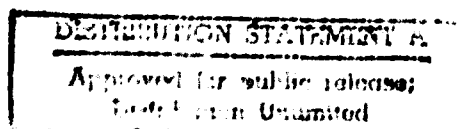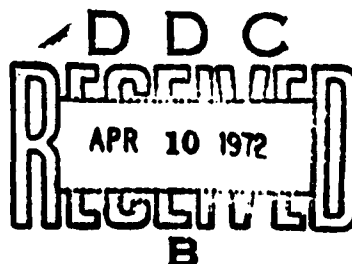# An Experiment on the Sensitivity of A Global Circulation Model: Studies in Climate Dynamics for Environmental Security

M. Warshaw and R. R. Rapp

A Report prepared for

ADVANCED RESEARCH PROJECTS AGENCY

DDC

RECEIVED

APR 10 1972

B

Rand
SANTA MONICA, CA 90406

35

ACCESSION for

CFSTI   WHITE SECTION ☐
DDC   BUFF SECTION ☐
UNANNOUNCED   ☐
JUSTIFICATION ....

....

BY
DISTRIBUTION/AVAILABILITY CODES

DIST.   AVAIL. and/or SPECIAL

DOCUMENT CONTROL DATA

1. ORIGINATING ACTIVITY	2a. REPORT SECURITY CLASSIFICATION
The Rand Corporation	UNCLASSIFIED
	2b. GROUP
	----

3. REPORT TITLE

AN EXPERIMENT ON THE SENSITIVITY OF A GLOBAL CIRCULATION MODEL: STUDIES IN CLIMATE DYNAMICS FOR ENVIRONMENTAL SECURITY

4. AUTHOR(S) (Last name, first name, initial)

Warshaw, M., R. R. Rapp

5. REPORT DATE	6a. TOTAL NO. OF PAGES	6b. NO. OF REFS.
February 1972	35	11

7. CONTRACT OR GRANT NO.	8. ORIGINATOR'S REPORT NO.
DAHC15 67 C 0141	R-908-ARPA

9a. AVAILABILITY/LIMITATION NOTICES	9b. SPONSORING AGENCY
DDC-A	Advanced Research Projects Agency

10. ABSTRACT

The results of a series of six 60-day integrations of the Mintz-Arakawa two-level, global-circulation model, designed to test the hypothesis that sizable changes in boundary conditions could be detected in the solutions even in the presence of uncertainties in the initial conditions. The variables considered are zonally averaged and time averaged values from the last 30 days of each 60-day run; the significance of the changes is tested by a classical analysis-of-variance technique. The model was run with a standard set of initial conditions and with two independent sets of random temperature variations superposed on the initial temperature field. With the ice of the Arctic Ocean replaced with water at the freezing point, the model was run once with the other standard initial conditions and once each with two sets of temperature variations. Changes in boundary conditions in the model are shown to produce detectable changes in the general circulation.

11. KEY WORDS

Atmosphere
Meteorology
Arctic
Radar
Computer Simulation

R-908-ARPA

February 1972

An Experiment on the Sensitivity of A Global Circulation Model: Studies in Climate Dynamics for Environmental Security

M. Warshaw and R. R. Rapp

A Report prepared for

ADVANCED RESEARCH PROJECTS AGENCY

Rand

SANTA MONICA, CA. 90406

PREFACE

Meteorological studies suggest that technologically feasible operations might trigger substantial changes in the climate over broad regions of the globe. Depending on their character, location, and scale, these changes might be both deleterious and irreversible. If a foreign power were to bring about such perturbations either overtly or covertly, either maliciously or heedlessly, the results might be seriously detrimental to the security and welfare of this country. So that the United States may react rationally and effectively to any such actions, it is essential that we have the capability to: (1) evaluate all consequences of a variety of possible actions that might modify the clim e, (2) detect trends in the global circulation that presage changes in the climate, either natural or artificial, and (3) determine, if possible, means to counter potentially deleterious climatic changes. Our possession of this capability would make incautious experimentation unnecessary, and would tend to deter malicious manipulation. To this end, the Advanced Research Projects Agency initiated a study of the dynamics of climate to evaluate the effect on climate of environmental perturbations. The present Report is a technical contribution to this larger study.

The ability to discern changes in the general circulation as simulated by a numerical model has been questioned because of the inherent unpredictability of the atmosphere. This study was undertaken to demonstrate that, despite the errors that inevitably occur in day-to-day predictions, the mean values are sensitive to major changes in the boundary conditions. A set of calculations has been made using the Mintz-Arakawa 2-level model to determine whether changes in the mean state can be found in the presence of error.

Reports on related work are: *The Heat Budget of the Arctic Basin and Its Relation to Climate*, by J. O. Fletcher, R-444-PR; *A Documentation of the Mintz-Arakawa Two-level Atmospheric General Circulation Model*, by W. L. Gates et al., R-877-ARPA; and *Global Turbidity Studies, I: Volcanic Dust Effects -- A Critical Survey*, by D. Diermendjian, R-886-ARPA.

SUMMARY

The growth of small errors in numerical models of the atmospheric circulation destroys the detailed predictive capability of those models within a few days. Despite the failure of the models to produce accurate local predictions, it was hypothesized that a change in the equator-to-pole temperature gradient would produce discernable effects in average conditions. This Report presents the results of an experiment to test this hypothesis.

The Mintz-Arakawa model was started with a standard set of initial conditions and was run 60 days. The experiment was then replicated twice, with two independent sets of random temperature variations superposed on the temperatures at the $\sigma = 0.25$ and $\sigma = 0.75$ levels. At this point, the ice of the Arctic Ocean was replaced with water at the freezing temperature. Again the model was run, starting once with the standard initial conditions (other than the ice removal) and once each with the two sets of temperature "errors" added.

For the four replications with temperature "errors," the detailed predictive capability was lost after about 14 days. Yet an analysis of variance applied to the last 30-day zonal-average values of the three sets of ice-in/ice-out runs showed many significant changes in the general circulation. These significant changes may be summarized as follows:

(1) The vertical stability over the Arctic Basin decreased markedly. An expected result of this decreased stability (and the newly available surface moisture) would be an increase in precipitation; analysis of this possibility will be deferred.

(2) The excess heat in the Arctic caused a slight efflux of mass from the area, which caused lower pressure over the Arctic Basin.

(3) The oceanic heat source present upon removal of the ice weakened the cold core vortex over the Arctic Basin.

(4) The geopotential heights at high levels in the sub-Arctic were increased by this change in the distribution of mass.

(5) The midlatitude westerlies were weakened by the redistribution of mass.

These results show that changes in boundary conditions in the model produce detectable changes in the general circulation despite the breakdown of predictability.

ACKNOWLEDGMENTS

CONTENTS

I. INTRODUCTION

In a recent paper, Lorenz (1969) raised a serious question regarding the ability to predict the future state of fluid systems. He states:

> ...that certain formally deterministic fluid
> systems which possess many scales of motion are
> observationally indistinguishable from indeter-
> ministic systems; specifically, that two states
> of the system differing initially by a small 'ob-
> servational error' will evolve into two states
> differing as greatly as randomly chosen states
> of the system within a finite interval, which
> cannot be lengthened by reducing the amplitude
> of the initial error.

Lorenz goes on to say:

> It is found that each scale of motion possesses
> an intrinsic finite range of predictability, pro-
> vided that the total energy of the system does not
> fall off too rapidly with decreasing wavelength.
> With the chosen values of the constants, 'cumulus-
> scale' motions can be predicted for about an hour,
> 'synoptic-scale' motions a few days, and the larg-
> est scales a few weeks in advance.

For those who propose using global-circulation models to investi-
gate purposeful and inadvertent climate modification the implications
of Lorenz's thesis would be serious indeed, if not disastrous. Errors
in initial conditions would tend to mask completely the desired effect
in a relatively short time. There are, however, some important differ-
ences between the primitive equation models and the idealized system
from which Lorenz reached his conclusions. These occur primarily in
the form of heating forcing functions and dissipative terms, both of
which are missing in the Lorenz formulation.

Our purpose is to investigate the possibility of detecting changes
in the mean conditions due to boundary value changes despite the fact
that predictability, in the Lorenz sense, has disappeared before the
period of averaging is complete.

In the absence of an analysis of error growth in the primitive
equations of motion, we look for other means to assess the fidelity
of current global circulation models. This report presents the results

of a series of six 60-day integrations of the Mintz-Arakawa two-level
global-circulation model, designed to test the hypothesis that the
presence of sizable changes   the boundary conditions could be deter-
mined in the solutions. The variables considered are values (zonally
averaged and time averaged) from the last 30 days of each 60-day run,
and the significance of the changes is tested by a classical analysis-
of-variance technique. The average differences between two arctic ice
conditions are then discussed in terms of qualitative hypotheses that
have been advanced to explain the circulation changes.

II. THE EXPERIMENT

It should be understood that the experiment described herein is one that estimates only the model's ability to separate signal (purposeful changes in boundary conditions) from noise (random changes in initial conditions). No claim can be made that the real atmosphere would behave in a similar way; one hopes that it would, but this is a separate problem and is not addressed here.

The experiment is easily described in a step-by-step manner. Our initial conditions were taken from day 370 of the Mintz-Arakawa model as run by UCLA. From this standard set of initial and boundary conditions for the model (Gates et al., 1971), an initial "control run" of sixty days was made. The first thirty days were to allow transients to settle and boundary conditions to make their effect felt. The last thirty days was the period over which the results were averaged. Next, this run was repeated exactly, but with a small random error added to the initial free-air temperature conditions at each grid point. The distribution from which this random error was drawn had zero mean and a variance of 1°C (less than the variance from actual measurements). These additive errors may be viewed as reflecting our uncertainty about global initial conditions if we were to attempt real prediction with the model.

After completing the second run, a third run exactly like the second was made, maintaining the same boundary conditions, but with a completely new set of random noise samples drawn from the same distribution as before. We now had three samples with the same boundary conditions and almost the same initial conditions. Whether the solutions as a function of time would be quite close to each other or significantly different was unknown. The extent to which these solutions are considered "close" depended on the outcome of the next series of trials described below.

At the beginning of Run 4, we introduced a change in the surface boundary conditions, discussed below. The first run with this new boundary condition was again called a control run and had no additive random noise contaminating the initial conditions. Runs 5 and 6 do have the

additive noise and this noise is <u>identical</u> to that which perturbed the
second and third runs with the original boundary condition.

III. CHANGING THE INITIAL CONDITIONS

We elected to perturb the initial temperature at the two interior levels of the model as the means of introducing noise into the initial conditions; these levels are identified in the model as $\sigma = 0.25$ and $\sigma = 0.75$, and are approximately the 400 and 800 mb surfaces, respectively. The temperature values at each grid point were modified by a number drawn from a normal distribution having zero mean and a variance corresponding at most to the variance encountered in actual temperature measurements. Drawing from a zero-mean population virtually assured that the post-perturbation global mean temperature would remain unchanged (given sufficient grid points).

The choice of magnitude of the variance of the noise was more difficult. If a large variance were chosen, the solutions might diverge drastically, leaving us to defend the challenge that the noise could realistically have been smaller with possibly a different outcome. If a small variance were chosen we would have been subject to challenge for not making a significant change. Of course, if very small perturbations had caused very different solutions, an important characteristic of the model would have been exposed. We chose the variance of the temperature perturbation to be 1°C. This is in accord with current GARP data specifications and has been used in other predictability experiments (Jastrow and Halem, 1970, Charney, 1966).

There are initial errors other than temperature that could affect the nature of the solutions, but our main objective was to ensure only that solutions with perturbations were different from control runs -- not to study the nature of the errors as such. In a later section we attempt to show that the temperature errors introduced did, indeed, produce the type of unpredictability discussed by Lorenz.

IV. CHANGING THE BOUNDARY CONDITIONS

The philosophy involved in choosing a change in boundary condition is considerably more subtle than that of the initial-condition perturbation. There is little scientific basis or prior experience to draw on. The purpose of the change is to induce a substantial change in the climate. In our case, "substantial" could mean just large enough to be perceived through the noisy initial conditions, or it could mean large enough to alter the human environment significantly somewhere on Earth, possibly distant from the immediate locale of the change.

Examples of major changes in boundary conditions could be the removal of all or a portion of the arctic ice, flooding the Sahara Desert, or radically changing the evaporation of, say, 10^6 sq km of low-latitude ocean. There was no experience which led us to believe with certainty that the model would respond to any of these changes with a substantial climatological change. They are "major" only because they represent very difficult or nearly impossible engineering feats for man.

There was no compelling reason to subject the system to such a severe change. An equally interesting approach would have been to try as small a change as one would guess was capable of producing an observable effect. Here the risk was that the effect would not be discernible.

In the experiments reported here we chose to alter the surface-boundary condition by removing all the sea ice in the northern hemisphere and replacing it with water at -1°C. In the Mintz-Arakawa model,[*] the ocean's surface temperature is prescribed, so once set at -1°C the arctic water temperature remains at that temperature.

---

[*] A complete documentation of the Mintz-Arakawa model appears in Gates et al., 1971.

V. PREDICTABILITY

A measure often used to estimate predictability is the root-mean-square difference between a control run and its perturbed counterpart. Let $T_{ij\sigma\ell}(t)$ be the temperature at grid point (i,j), sigma level $\sigma = 0.25,\ 0.75$, run number ℓ and time t. The run number, $\ell = 1,2,\ldots,6$, indexes the simulation as defined in Table 1.

Table 1

ℓ-INDEXING OF EXPERIMENTS

ℓ	Experiment Description
1	Ice-in control
2	Ice-out control
3	Ice-in perturbation no. 1
4	Ice-out perturbation no. 1
5	Ice-in perturbation no. 2
6	Ice-out perturbation no. 2

Then predictability in terms of temperature is measured by

$$\Delta T_\sigma(t) = \left\{ \frac{1}{N} \sum_{\text{all } i,j}^{N} [T_{ij\sigma\alpha}(t) - T_{ij\sigma\beta}(t)]^2 \right\}^{1/2} \qquad (1)$$

where $\beta = 3$ or 5 if $\alpha = 1$ and $\beta = 4$ or 6 if $\alpha = 2$. Figures 1 to 4 show $\Delta T_\sigma(t)$ for $\sigma = 0.25$ and 0.75 and $\alpha = 1$ and 2.

In all cases the value of $\Delta T_\sigma(t)$ dropped to about 0.5°C in two days after the perturbations were introduced (at day 370). This is most likely due to the smoothing out of the initial random errors which were introduced. From day 372 the errors increase in a rather regular fashion until about day 390. From then until the end of the run, $\Delta T_\sigma(t)$ oscillates erratically with an amplitude of about 2°C. We take this to indicate that the systems reached states where the predictability has essentially been lost. We therefore assume that any statistically significant differences between means over the last thirty days of the runs must be the effect of changing the boundary conditions.

Fig. 1 -- Global root-mean-square temperature deviation as a
function of time: ice in; σ = 0.75.

Fig. 2 -- Global root-mean-square temperature deviation as a
function of time: Ice in; σ = 0.25.
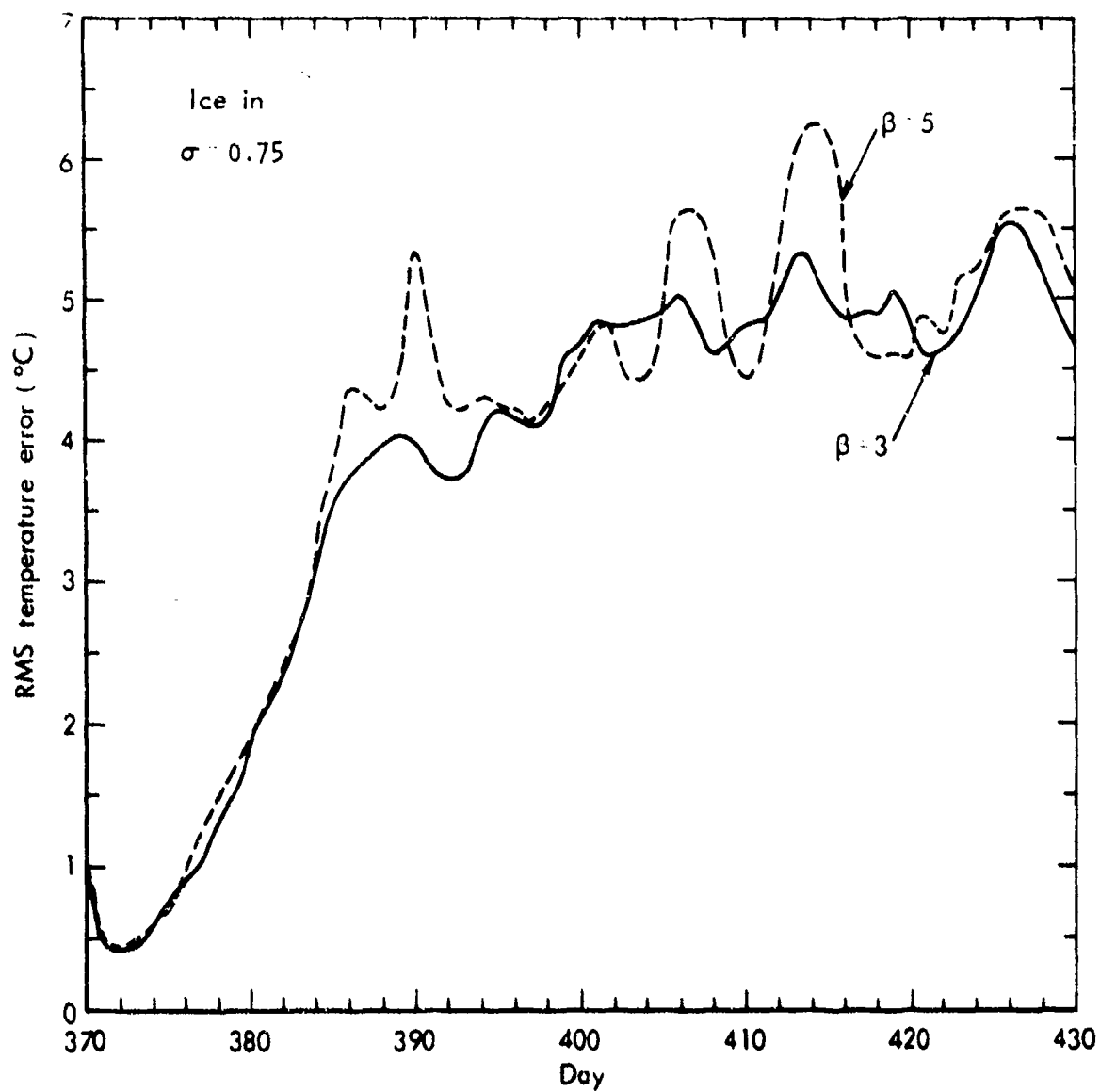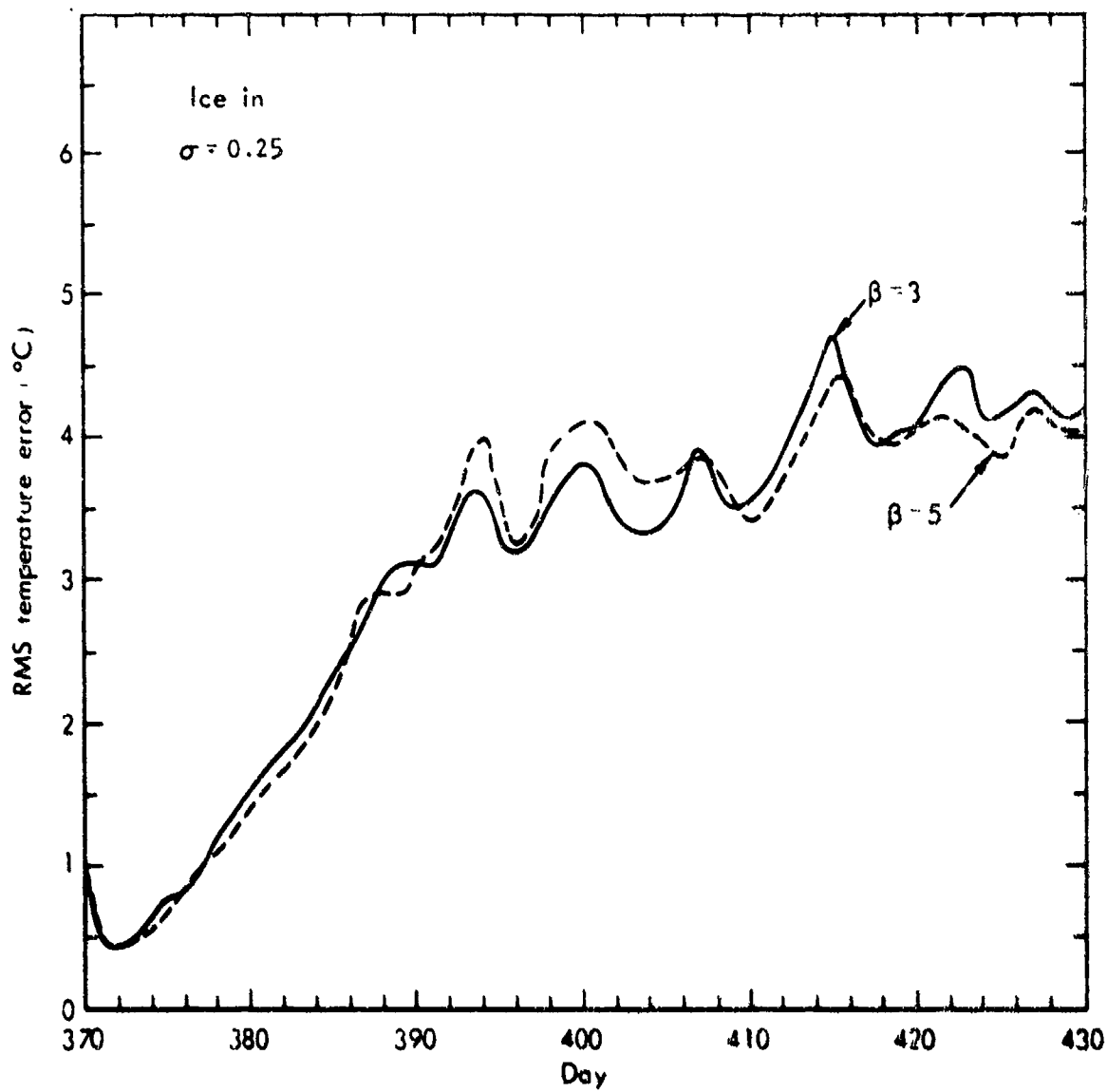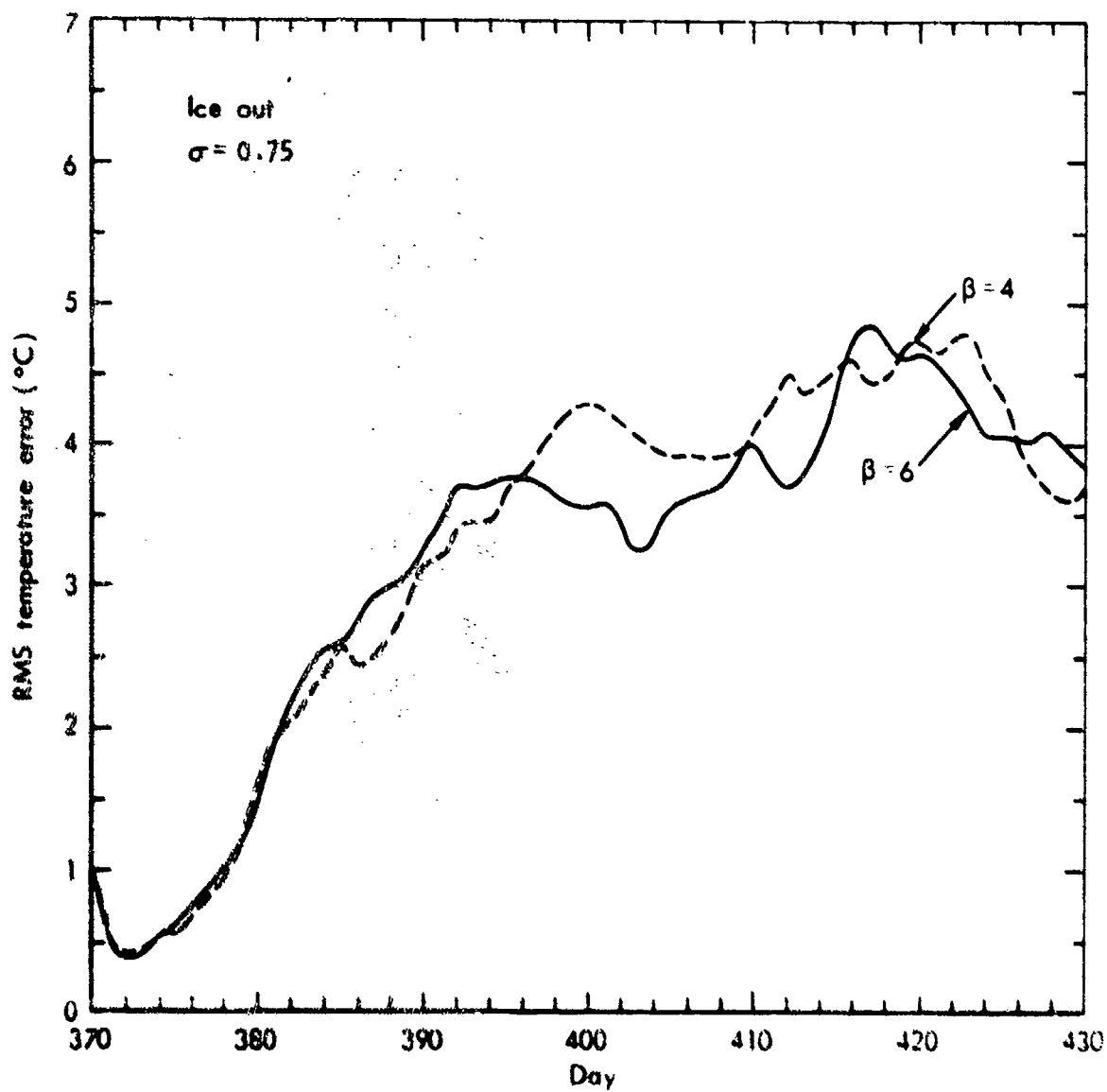
Fig. 3 -- Global root-mean-square temperature deviation as a
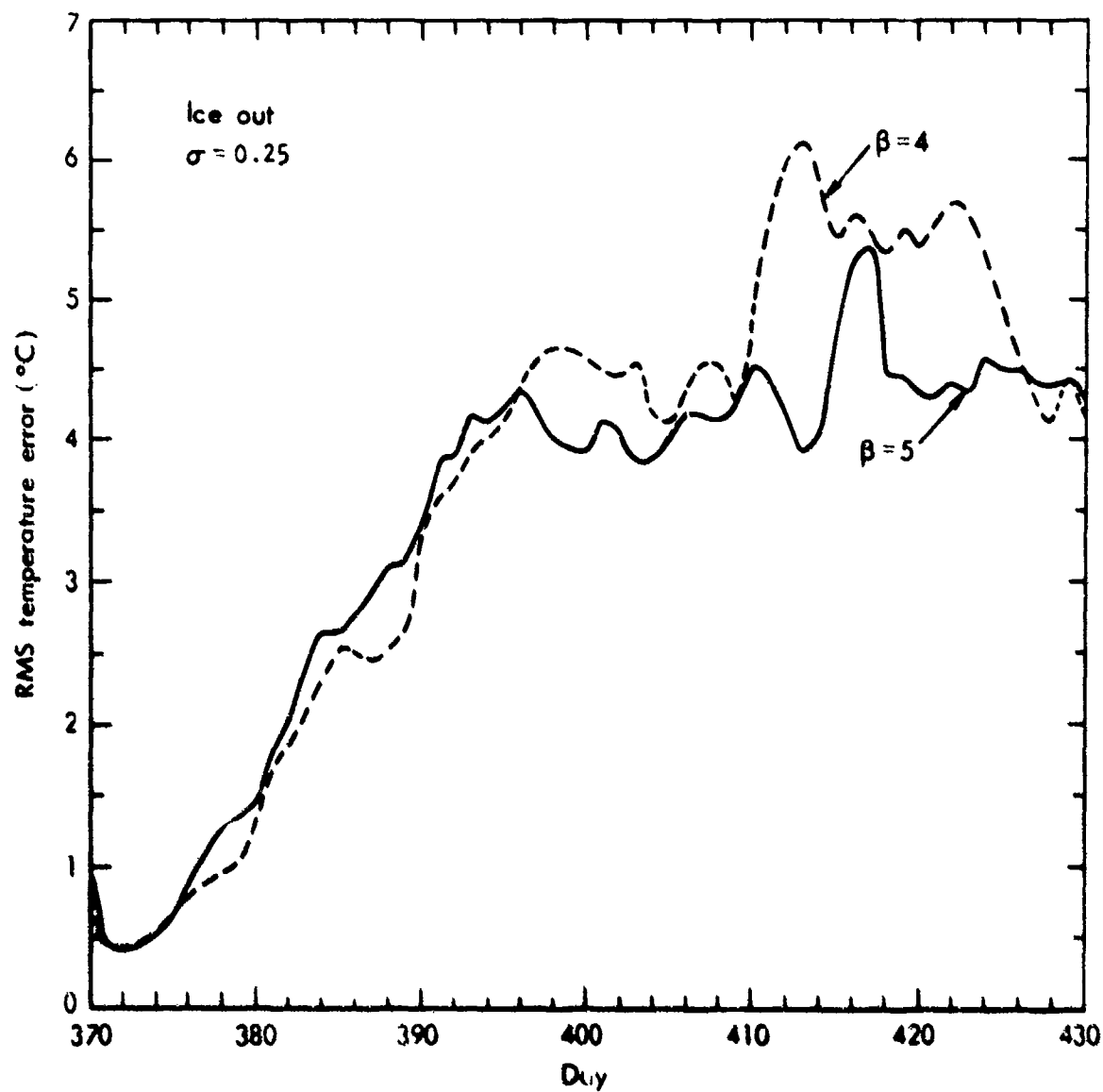function of time: ice out; σ = 0.75.

Fig. 4 -- Global root-mean-square temperature deviation as a
function of time: ice out; σ = 0.25.

VI. STATISTICAL TESTS

The question we ask is whether or not the change in boundary con-
dition produces a significant change in some climatic variable of in-
terest. This could have _failed_ to occur for several reasons: (1) the
random perturbations in initial conditions produced inordinately large
changes in the final climatic variables, thus totally obscuring the
actual effect of different boundary conditions, (2) the boundary con-
dition change was not large enough to produce an effect in the allotted
simulation time; or (3) the change in boundary condition was not physi-
cally significant, i.e., it was either a wrong choice for affecting fu-
ture climate, or the model did not properly represent the physics. A
straightforward analysis-of-variance procedure may be used to test the
hypotheses that the boundary-condition change had no effect, or that
the additive noise had no effect.

For all variables which are averaged over the last 30 days and over
all values of longitude at a fixed latitude (the "30-day zonal mean"
the statistical procedure reduces to a simple univariate analysis of
variance. By such an averaging procedure all data were reduced, in our
experiment, to a 2 × 3 array of _scalar_ variables shown in Table 2.

Table 2

ARRAY OF SCALAR VARIABLES

	Unperturbed	Perturbation No. 1	Perturbation No. 2	
Ice In	Y_{11}	Y_{12}	Y_{13}	$Y_{1.}$
Ice Out	Y_{21}	Y_{22}	Y_{23}	$Y_{2.}$
	$Y_{.1}$	$Y_{.2}$	$Y_{.3}$	

where Y_{ij} is the zonally averaged 30-day mean of the variable of interest.

By assuming that the Y_{ij} are independent, normally distributed ran-
dom variables, we can test the hypothesis that the row means ($Y_{i.}$) or the
column means ($Y_{.j}$) in Table 2 come from the same distribution. Referring

to Table 1 we see that rejecting the null hypothesis that all row means come from the same distribution is the same as rejecting the hypothesis that removing the arctic ice had <u>no</u> effect on the physical variable under observation. Similarly, rejecting the null hypothesis that all column means come from the same distribution is the same as rejecting the hypothesis that perturbing the initial conditions had no effect on the variable.

The methods available allow us to guard only against type I errors. That is, when the null hypothesis is true, the probability of rejecting it can be made as small as we wish.

We cannot, however, make any statement about the probability of accepting the null hypothesis when, in fact, it is false (a Type II error). This is the less objectionable of the two types of error, and while an estimate of its probability would be useful, it is by no means crucial.

A straightforward treatment of the analysis of variance may be found in most texts on mathematical statistics (e.g., Hoel, 1947; Rao, 1965). Adopting the notation

$$Y_{..} = \frac{1}{ab} \sum_{i,j} Y_{ij} \tag{2}$$

$$Y_{.j} = \frac{1}{a} \sum_{i=1}^{a} Y_{ij} \tag{3}$$

$$Y_{i.} = \frac{1}{b} \sum_{j=1}^{b} Y_{ij} \tag{4}$$

where a = number of rows and b = number of columns, let

$$A = \sum_{i=1}^{a} \sum_{j=1}^{b} (Y_{ij} - Y_{.j} - Y_{i.} + Y_{..})^2 \tag{5}$$

$$B = a \sum_{j=1}^{b} (Y_{.j} - Y_{..})^2 \tag{6}$$

$$C = b \sum_{i=1}^{a} (Y_{i.} - Y_{..})^2 \tag{7}$$

We may now make two hypotheses about the outcome of the experiment and, from Eqs. (5) through (7), form statistics with which to test the hypotheses.

Hypothesis I: <u>Removing the ice had no effect</u> ($\equiv$ the row means $(Y_{i.})$ are samples from the same distribution). Let

$$f = \frac{C(a - 1)(b - 1)}{A(a - 1)} \tag{8}$$

Then f has an F-distribution with $[a - 1, (a - 1)(b - 1)]$ degrees of freedom. A test of Hypothesis I is based on the following reasoning.

Let H_0 represent the hypothesis under consideration and define the conditional probability:

$$\alpha = P_r(\text{reject } H_0 | H_0 \text{ is true})$$

Next find the value F* for which

$$\int_{-\infty}^{F^*} F(\xi) d\xi = 1 - \alpha = P_r(\text{accept } H_0 | H_0 \text{ is true})$$

Now if $f \leq F^*$ we accept the hypothesis H_0 and if $f > F^*$ we reject H_0. F* is found in any table of the F-distribution as a function of α and the degrees of freedom $[v_1, v_2]$.

Hypothesis II: <u>Perturbing the initial conditions with additive random noise had no effect</u> ($\equiv$ the column means $(Y_{.j})$ are samples from the same distribution). Then

$$F = \frac{B(a - 1)(b - 1)}{A(b - 1)}$$

and f has $[b - 1, (a - 1)(b - 1)]$ degrees of freedom.

TEMPE°ATURE, GEOPOTENTIAL HEIGHT, AND ZONAL WIND

The analysis-of-variance test described above was applied first to the temperature, the geopotential height, and the zonal (E-W) wind velocity. Our goal was to ascertain whether there is a significant difference in these variables as a function of altitude and latitude when the arctic sea ice was removed.

Altitude is here taken to be the 1000 mb, 800 mb, and 400 mb pressure levels. Latitude is usually defined as an annular region over which the variable has been averaged. For instance, we tested the value of the average temperature between 54°N and 70°N latitude at the 800 mb level. The raw data going into the average were the values of this variable at all the grid points provided by the simulation lying within the specified annular region.

Gradients of variables have been specified at exact latitudes and have no meridional averaging.

Table 3 gives all the pertinent data: the variable, meridional region, altitude level, value of the variable for all 6 experiments (3 ice-in and 3 ice-out), difference of ice-in and ice-out mean values, and finally, the value of the F-statistic and its significance level. We choose to accept the hypothesis that there is no difference between the ice-in and ice-out results if the significance is no better than 0.05. Therefore we reject the hypothesis of no differences if and only if the significance level > 0.05.

MERIDIONAL TRANSPORT

In addition to the basic variables of temperature, geopotential height, and zonal wind, we selected the meridional transport of sensible heat and momentum for testing. Again, these derived quantities were the 30-day mean, zonally averaged values. Their derivation, utilizing the available Mintz-Arakawa data, is given below:

Heat transport. The meridional sensible-heat flux is given by $f = \rho c_p vT$, where ρ = density of air, c_p = specific heat of air = 0.24 cal $gm^{-1} deg^{-1}$.

Table 3

DATA AND STATISTICS FOR TEMPERATURE, GEOPOTENTIAL HEIGHT, ZONAL WIND, POLAR HEAT TRANSPORT, AND POLAR MOMENTUM TRANSPORT

Variable	Region	Altitude (mb)	Ice-In Control	Ice-In Pert. 1	Ice-In Pert. 2	Ice-Out Control	Ice-Out Pert. 1	Ice-Out Pert. 2	Mean Ice-In/Out Difference	F-Statistic (1,2) Deg Freedom	Significance
Temperature, T, °C	70-40°N	1000	-21.681	-23.185	-21.987	-6.044	-5.382	-6.145	-15.765	189	Reject P .005
		800	-26.432	-26.624	-26.61	-17.284	-16.155	-17.342	-8.970	95	Reject P .01
		400	-48.884	-50.279	-47.277	-53.084	-49.974	-52.166	2.929	3.23	Accept
	50-70°N	1000	-11.823	-11.121	-11.034	-6.561	-6.021	-6.086	-5.104	3169	Reject P << .005
		800	-19.101	-18.916	-18.214	-14.218	-14.034	-14.008	-4.658	432	Reject P << .005
		400	-45.866	-47.449	-45.943	-46.185	-46.142	-45.809	-0.339	4.67	Accept
	30-50°N	1000	4.184	4.701	4.072	4.525	4.606	4.84	-0.338	1.82	Accept
		800	-5.230	-4.745	-5.229	-1.703	-0.762	-4.472	-1.423	1.74	Accept
		400	-33.261	-34.112	-32.583	-33.153	-32.756	-32.785	0.622	5.82	Accept
Temperature Gradient ΔT	40°N	T(1000)-T(400)	25.949	25.893	27.448	52.308	48.842	52.621	-24.869	619	Reject P << .005
Geopotential Height	70-40°N	1000	4.705	4.667	4.731	4.404	4.473	4.722	0.302	23.5	Reject P .05
		800	20.941	20.944	21.126	21.441	21.532	21.445	-0.466	35.3	Reject P .025
		400	68.607	68.491	69.202	69.391	69.840	69.472	-0.788	6.38	Accept
	50-70°N	1000	6.090	6.413	6.296	6.220	6.213	6.221	0.06	14.5	Accept
		800	22.970	22.937	23.039	23.279	23.291	23.285	-0.303	110	Reject P .01
		400	71.008	71.265	71.195	71.739	71.737	71.771	-0.760	43.5	Reject P .025
Geopotential Height Gradient ΔH	H(86)-H(66)	800	-3.756	-3.592	-3.58	-3.492	-3.470	-3.565	-0.134	1.43	Accept
		400	-3.370	-2.815	-2.775	-3.230	-2.900	-3.481	0.217	0.74	Accept
	H(66)-H(46)	800	-1.322	-1.456	-1.199	-0.948	-0.990	-0.981	-0.353	23.7	Reject P .05
		400	-1.517	-1.647	-3.110	-1.404	-2.669	-2.545	-1.014	18.1	Reject P .05
Zonal Wind Velocity, V, m/sec	70-40°N	800	-0.171	-1.917	4.220	0.805	1.347	1.576	-1.872	6.86	Reject P .05
		400	3.708	0.896	2.673	5.580	4.697	6.794	-3.265	21.6	Reject P .05
	46-62°N	800	7.799	9.790	7.434	5.911	7.278	6.816	1.670	9.0	Accept
		400	18.639	21.458	17.508	14.177	16.282	15.580	3.856	15.3	Reject P .05
Polar Heat Transport, F	70°N	800	2.308×10^{14}	2.668×10^{14}	3.164×10^{14}	1.306×10^{14}	1.324×10^{14}	1.055×10^{14}	1.485×10^{14}	20.6	Reject @ .05
		400	1.012×10^{14}	-1.346×10^{14}	-1.146×10^{14}	0.190×10^{14}	0.770×10^{14}	-5.484×10^{14}	1.015×10^{14}	0.53	Accept
Polar Momentum Transport, M	70°N	total	1.265×10^{26}	1.443×10^{26}	1.356×10^{26}	0.771×10^{26}	0.639×10^{26}	0.477×10^{26}	0.522×10^{26}	11.3	Accept

v = northward wind speed, and T = temperature. Integrating over a rectangular region in the $x - z$ plane gives the total heat flux, F, as

$$F = \int_{x_0}^{x_1} \int_{z_{surface}}^{z_{max}} f \, dz \, dx \qquad (9)$$

Upon use of the hydrostatic approximation $dp = -\rho g \, dz$, the definition of a σ-surface $dp = (p_s - p_0) \, d\sigma$, and the transformation to polar coordinates $dx = R \cos \phi \, d\lambda$, this becomes

$$F = \frac{c_p R \cos \phi}{g} \int_0^{2\pi} \int_0^1 (p_s - p_0) vT \, d\sigma \, d\lambda \qquad (10)$$

where g = acceleration due to gravity = 981 cm sec^{-2}, p_s = surface pressure, p_0 = pressure at top of model atmosphere = 200 mb, σ = sigma surface $(0 \leq \sigma \leq 1)$, R = radius of earth = 6.375×10^8 cm, ϕ = latitude, and λ = longitude.

To utilize the Mintz-Arakawa model's data, which are given only at σ = 0.25 and 0.75, we assume that the mean values of v and T in the interval $0 \leq \sigma \leq 0.5$ are given by the values at σ = 0.25. Similarly the mean values of v and T in the region $0.5 \leq \sigma \leq 1.0$ are given by those at σ = 0.75.

The vertical integration is then approximated by

$$F = \frac{c_p R \cos \phi}{2g} \int_0^{2\pi} (p_s - p_0)(v_{.25}T_{.25} + v_{.75}T_{.75}) \, d\lambda \qquad (11)$$

With $d\lambda = 2\pi/72$, where 72 is the number of zonal grid increments in the model, the sum approximating the meridional heat transport is

$$F = \frac{c_p R \cos \phi}{72g} \sum_{i=1}^{72} \left\langle (p_s - p_0)_i (v_{.25}T_{.25} + v_{.75}T_{.75})_i \right\rangle \qquad (12)$$

where $\langle ... \rangle$ denotes the 30-day averaging operation.

In practice we split the integration shown in Eq. (12) into two parts, the lower level given by the $V_{.75}T_{.75}$ term and the upper level given by the $V_{.25}T_{.25}$ term. They may be added if the total heat transport is desired. The reason for the partitioning is given in the next section, but the values for polar heat transport for each level at 70°N for the six cases defined previously are given in Table 3.

The F-statistics produced by an analysis-of-variance test indicated a significant difference between the ice-in and the ice-out heat transport in the lower level. (We reject the hypothesis of equality at the 0.05 level).

There was no significant difference in meridional heat transport between ice-in and ice-out at the upper level.

Meridional momentum transport. In a similar manner we proceeded to evaluate the meridional momentum transport and test for a significant change caused by the removal of polar ice.

From Hess (1959), the total poleward momentum transport at a fixed latitude, ϕ, was given by

$$M = -\int_{x_0}^{x_1} \int_{z_{surface}}^{z_{max}} \rho m v \, dz \, dx \tag{13}$$

where m = momentum and v = northward wind speed. Making the substitutions $dx = R \cos \phi \, d\lambda$, $m = (v + \Omega R \cos \phi) R \cos \phi$, $dp = -\rho g \, dz$, we found

$$M = \frac{R^2 \cos^2 \phi}{g} \int_0^{2\pi} \int_{p_s}^{p_0} (U + \Omega R \cos \phi) v \, dp \, d\lambda \tag{14}$$

where v = eastward component of wind, and Ω = angular velocity of rotation of the earth = 7.272×10^{-5} sec^{-1}.

We next transform to σ-coordinates, obtaining

$$M = \frac{-R^2 \cos^2 \phi}{g} \int_0^{2\pi} \int_0^1 (p_s - p_0)(U + \Omega R \cos \phi) v \, d\sigma \, d\lambda \quad (15)$$

Again, taking the values at $\sigma = 0.25$ and $\sigma = 0.75$ as the mean values in the layers $0 \leq \sigma \leq 0.5$ and $0.5 \leq \sigma \leq 1$ we found,

$$M = \frac{-R^2 \cos^2 \phi}{2g} \int_0^{2\pi} (p_s - p_0) \left[(U_{.25} + \Omega R \cos \phi) v_{.25} \right.$$

$$\left. + (U_{.75} + \Omega R \cos \phi) v_{.75} \right] d\lambda \quad (16)$$

or

$$M = \frac{-\pi R^2 \cos^2 \phi}{72g} \sum_{i=1}^{72} \left\langle (p_s - p_0)_i \left[U_{.25} v_{.25} + U_{.75} v_{.75} \right. \right.$$

$$\left. \left. + \Omega R \cos \phi (v_{.25} + v_{.75}) \right] \right\rangle \quad (17)$$

Table 3 shows the polar momentum transport at 70°N latitude. The appropriate F-test led us to conclude that there is no significant difference in momentum transport upon removal of the arctic ice.

Briefly summing up, we tested a selection of meteorological variables to ascertain if their value had changed significantly after 60 days as a result of removing the arctic sea ice. The "Significance" column of Table 3 indicates that some variables changed and some did not. Note that we have not estimated a confidence level for the magnitude of the change, nor have we discussed steady-state asymptotic differences. Section VIII discusses the physical implications of these findings.

VII. PHYSICAL INTERPRETATION

The arctic pack ice has long been thought to be a controlling fac-
tor in the climate of the globe. Budyko (1962) theorized that it would
require only small changes in the albedo of the ice to cause its de-
struction within a few years. Fletcher (1965) pointed out that the
ice acts in two ways to keep the arctic atmosphere cold: first, it re-
flects much of the incident solar radiation in the summer; and, second,
it prevents heat from the ocean from being transferred to the atmosphere
during the winter. Fletcher further pointed out that removal would lead
to much warmer temperatures over the Arctic Basin in the winter but only
slightly warmer temperatures during the summer. Moreover, an open Arc-
tic Ocean would be a source of moisture for the atmosphere that would
tend to increase the precipitation in the Arctic.

In regard to changes in the circulation of the atmosphere, Fletcher
argued that a winter decrease in the pole-to-equator temperature gradi-
ent would cause a weakening of the westerly circulation in winter.
Pogosian (1970) attempted to quantify this argument by simply assuming
warmer surface temperatures and constructing maps of the baric topo-
graphy at high altitudes from simple hydrostatic considerations. The
experiment reported here, carefully designed to eliminate any differ-
ences not attributable to the removal of the ice, indicated that the
model behaves much in the fashion that has been postulated for the real
atmosphere in winter.

The most obvious and most significant difference was in the 1000 mb
temperature over the Arctic Ocean. This increase when the ice was re-
moved is also significant at the 800 mb level, but there was an insig-
nificant cooling at the 400 mb level. Taken together they indicated a
significant steepening of the lapse rate over the Arctic. Figure 5
shows the model temperatures on a tephigram together with the mean
vertical temperature sounding for February as reported by Fletcher
(1966b) for the North Pole 4 ice station during 1955-56. The differ-
ence in lapse rate is to be expected because with an open ocean there
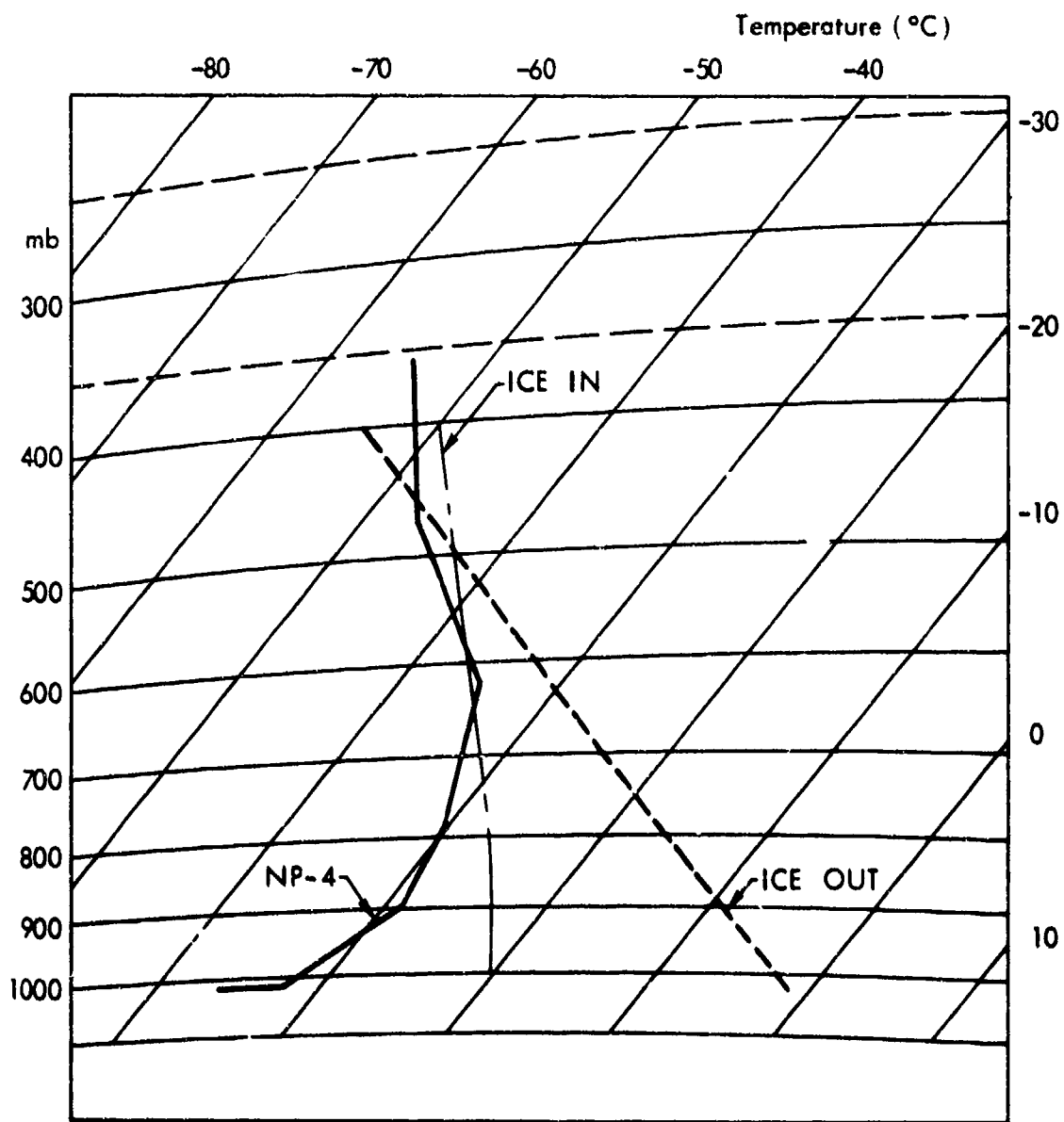is a strong heat source at the surface.

Fig. 5 -- Model temperatures on a tephigram together with mean vertical temperature sounding for February for the North Pole 4 ice station during 1955-56 (Fletcher, 1966b).

The picture of the meridional and vertical distribution of tem-
perature difference is shown in Fig. 6. This figure was constructed
by forming the mean monthly zonally averaged temperature for each lati-
tude in the grid for each level for all six runs. The three ice-in
runs were then averaged and subtracted from the average of the three
ice-out runs. The striking feature of this chart is the southward ex-
tent of the warming into sub-Arctic latitudes. The warming was found
to be significant in the latitude band from 54 to 70°N at both the
1000 and 800 mb surfaces. This change was brought about because the
outbreaks of polar air which normally cool the sub-Arctic regions were
not as cold with an ice-free Arctic as they would have been with an
ice-covered Arctic. This explanation was further supported by the
significantly lower heat transport under ? -out conditions, across
70°N in the lower layer of the model atmosphere. The changes in tem-
perature at the 400 mb level and south oi 54°N were too small to be
ascribed to the removal of the ice ı this experiment.

Figure 7 shows the changes in the geopotential height for the
three levels; it was constructed in the same manner as Fig. 6. The
most striking feature of this chart is the <u>lowering</u> of the geopotential
height in the lower layers of the Arctic. If there were a strictly
hydrostatic change, these heights would have been expected to rise at
all levels with very little rise at the bottom and increasing lifting
as altitude increased. The sinking of the geopotential heights at the
low altitudes must have resulted from a dynamic change that removed
mass from the Arctic Region. This rather small change in mass over
the Arctic was not noticed as an increase in surface pressure elsewhere,
because it was spread over a much larger area as it was displaced south-
ward, but it had a significant effect on the dynamics of the circulation.
The two lower levels were significantly lower over the Arctic, and the
two higher levels were significantly higher over the sub-Arctic. Thin
dynamic redistribution of mass should have the effect of increasing the
west-to-east circulation at low levels in the Arctic, while decreasing
the west-to-east circulation at high levels in the sub-Arctic. A test
of the difference in geopotential between 46 and 66°N showed a signifi-
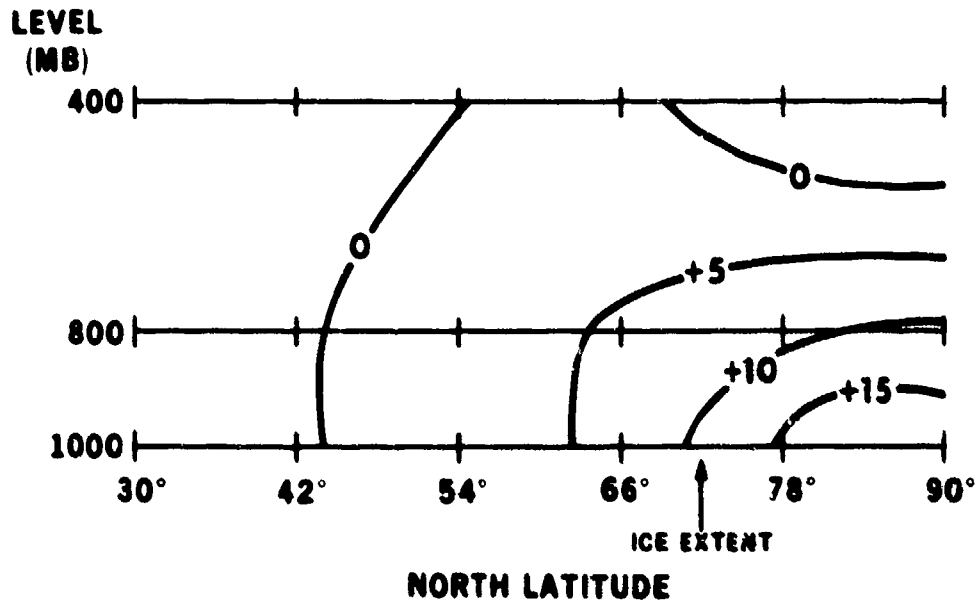cant decrease in the gradient at 800 and 400 mb.

LEVEL
(MB)

Fig. 6 -- Temperature differences
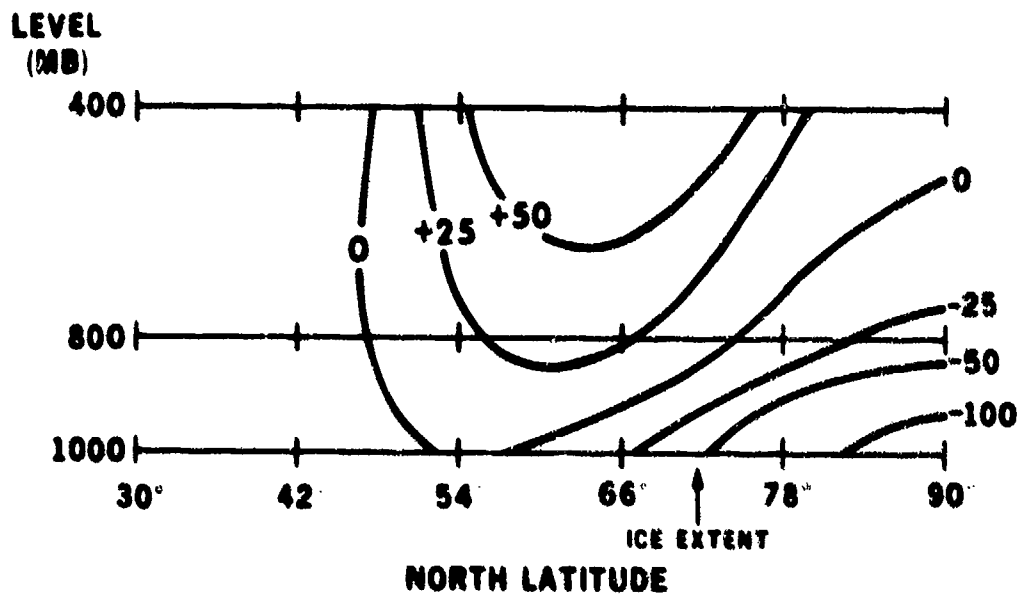(°C); (ice out) - (ice in).

LEVEL
(MB)

Fig. 7 -- Geopotential-height differences
(m); (ice out) - (ice in).

Figure 8 presents a cross section of the zonal wind velocity constructed in the manner of the two previous figures. The decrease of 4 m/sec at the 400 mb level at 54°N is significant and represents a confirmation of Fletcher's hypothesis for a weaker circulation in the winter. The significant 2 m/sec increase at 78°N had not been predicted in any of the qualitative studies of the problem. Although the statistical tests failed to show that the differences in the wind at the 800 mb level were significant, the significant changes in the pressure gradient responsible for the increase at 800 mb lent credence to the reality of the change. Another bit of evidence of the weakening of the sub-Arctic upper-level circulation was the change in the transport of angular momentum into the Arctic. Although the statistical test failed to reach the 5 percent significance level, the momentum flux was cut in half, indicating a decrease in the momentum available to be transported northward.
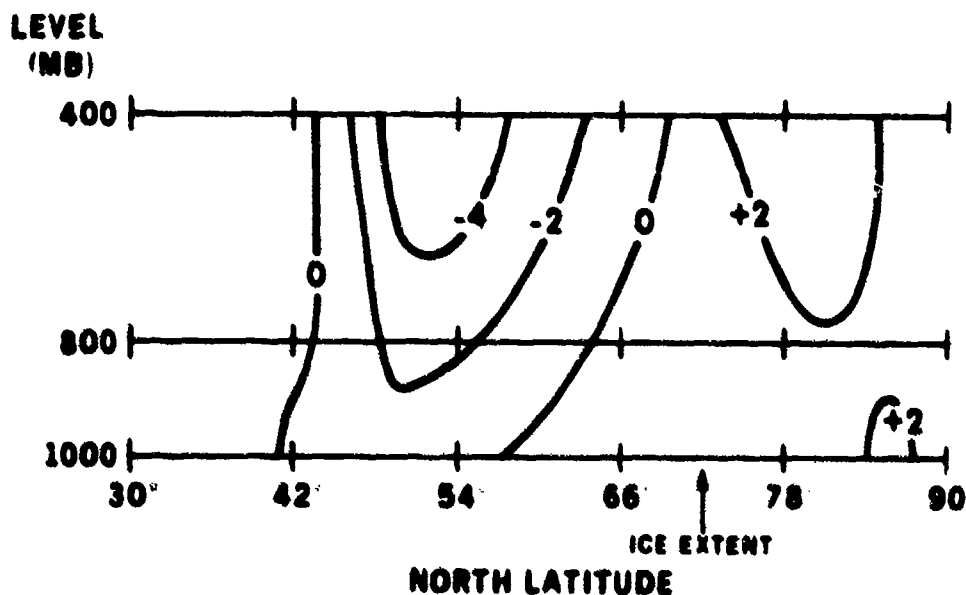

Fig. 8 -- East/west wind differences
(m/sec); (ice out) - (ice in).

VIII. SUMMATION

The changes in the model behavior between ice-in and ice-out, which have been shown by the statistical tests to be the results of the change in ice conditions, can be summarized as follows:

(1) The vertical stability over the Arctic Basin decreased markedly. An expected result of this decreased stability would be an increase in precipitation; analysis of the precipitation will be deferred.

(2) The excess heat in the Arctic caused a slight efflux of mass from the area, which caused lower pressure over the Arctic Basin.

(3) The oceanic heat source present upon removal of the ice weakened the cold core vortex over the Arctic Basin.

(4) The geopotential heights at high altitudes in the sub-Arctic were increased by this change in the distribution of mass.

(5) The mid-latitude westerlies were weakened by the re-distribution of mass.

This paper has thus indicated that changes of the boundary conditions in the model produce detectable changes in the output despite the breakdown of predictability. We believe that we have provided an adequate demonstration of the response of a primitive equation model to a rather major change in the boundary conditions. The relation between model response and the possible response of the real atmosphere is suggestive, but further work needs to be done to relate the model to the reactions of the atmosphere. In perusing the output of the six runs made to complete this experiment, we became acutely aware of the need for strict statistical controls. The natural variability of the model, and certainly the natural variability of the real atmosphere, must also be taken into account in trying to assess the effect of any purposeful or inadvertent changes in the factors which are suspected of controlling the climate.

REFERENCES

Budyko, M. I. (1962): "Polar Ice and Climate," *Bull. Acad. Sci. USSR*, No. 6, pp. 3-10 [translated in Fletcher (1966a), pp. 3-21].

Charney, J. G. (1966): 'The Feasibility of a Global Observation and Analysis Experiment," Publication No. 1290, Nat. Acad. Sci., Nat. Res. Council, Washington, D. C., pp. 3-31.

Fletcher, J. O. (1965): *The Heat Budget of the Arctic Basin and Its Relation to Climate*. R-444-PR, The Rand Corporation, Santa Monica, California.

Fletcher, J. O., B. Keller, and S. M. Olenicoff (eds.) (1966a): *Soviet Data on the Arctic Heat Budget and Its Climatic Influence*, RM-5003-PR, The Rand Corporation, Santa Monica, California.

Fletcher, J. O. (ed.) (1966b): *Proceedings of the Symposium on the Arctic Heat Budget and Atmospheric Circulation*, RM-5233-NSF, The Rand Corporation, Santa Monica, California.

Gates, W. L., E. S. Batten, A. B. Kahle, and A. B. Nelson (1971): *A Documentation of the Two-Level Two-Level Atmospheric General Circulation Model*, R-877-ARPA, The Rand Corporation, Santa Monica, California.

Hess, S. L. (1959): *Introduction to Theoretical Meteorology*, Holt, Rinehart & Winston, New York.

Hoel, P. G. (1947): *Introduction to Mathematical Statistics*, John Wiley and Sons, New York.

Jastrow, R., and M. Halem (1970): "Simulation Studies Related to GARP," *Bull. Am. Meteorol. Soc.*, **51**, 6, pp. 490-513.

Lorenz, E. N. (1969): "The Predictability of a Flow which Possesses Many Scales of Motion," *Tellus*, **21**, 3, pp. 289-307.

Pogosian, Kh. P. (1970): "If the Polar Ice were Annihilated...," *Priroda*, No. 6, pp. 74-79.

Rao, C. R. (1965): *Linear Statistical Inference and Its Applications*, John Wiley and Sons, New York.